Non-profits and the Profit Distribution Constraint with Selfish Entrepreneurial Motivations

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Non-profits and the Profit Non-distribution Constraint with Selfish Entrepreneurial Motivations

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Abstract
The profit non-distribution constraint (NDC) is the basic requirement for non-profit tax privileges all over the world. In donative non-profits it is justified by the need to prevent donors’ exploitation by firm owners. Donative non-profits, however, are not the only type of third-sector institution and in many of them the chief stakeholders are customers, not donors. In this paper we investigate the social value of the NDC in mutual non-profits where customers exert control over the firm. The main theme here is the exploitation of some customers by others, rather than the exploitation of donors by managers or owners. When customers are homogeneous, the NDC can never be beneficial. More complex is the case of customer heterogeneity. We show that in this case the NDC presents three issues. First, there are cases where intra-member exploitation occurs through the generation of accounting losses, and in such cases the NDC is ineffective from an allocative standpoint. Second, even when it prevents exploitation, the resulting allocations may be socially inferior to those that obtain in the presence of minority exploitation. We further show that non-members’ exploitation is of the same nature as minority members’ exploitation and it is just a variant of it, whereby the same results on the NDC’s effectiveness qualitatively hold in this case too. The upshot is that granting tax relief to all mutual non-profits adopting the NDC is unwarranted and a selective application of tax incentives based on the governance structure is instead needed.

Keywords
Non-profits; Non-distribution constraint; Customer ownership; Non-profits taxation.

JEL codes
L31; P13; D21; L20 G32

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1 Introduction

An eye-catching fact about non-profit institutions is their deep diversity in aims, operational modes, and legal forms. There is, however, one feature that keeps them all together—the prohibition to distribute profits to certain stakeholders. Though varying across countries, and sometimes even across different institution types within the same country, the provision always includes a ban on surplus transfers to the people who are in control of the organization, in particular shareholders, members and managers. This is the core of what is generally known as the profit non-distribution constraint (hereinafter NDC). A further observation is that everywhere the NDC affords the adopting organization the right to tax breaks. In this regard too we have an ample variety of arrangements across countries and non-profit types, such as exemptions from corporate income tax, donor charitable deductions, and many others (to get an idea of how wide and diverse the scope of NPs’ tax privileges is, see for example Fremont-Smith, 2004). Today, then, non-profits are virtually everywhere characterized by a combination of NDC and tax privileges attached to it.

Preferential tax treatment of non-profits has long been a subject of debate both in economics and law. Is it justified and, if so, in which forms? A part of the literature takes tax relief to non-profits for granted and just debates the appropriateness of specific tax instruments and eligibility requirements, with a view to improving their effectiveness (Bittker and Rahdert, 1976; Hansmann, 1981; Malani and Posner, 2007). Another part of the literature instead calls into question favouring non-profits through preferential tax regimes. Criticism hinges around the distortions caused by tax breaks, which roughly fall into two categories: those affecting market competition and allocation, and those affecting the internal efficiency of the firm. Bolton and Mehran (2006) show how competition between for-profits and non-profits can be distorted by tax privileges granted to NPs. Other papers, like Core et al. (2006) and Sansing and Yetman (2006), show that tax breaks can exacerbate agency problems and thus have negative effects on internal efficiency. All these contributions deal with donative non-profits whose main income source is donations. Here we pursue the same aim of this literature—probing preferential tax regimes—but our focus is on the foundations of tax relief rather than on specific instruments. Tax breaks for non-profits are always conditioned to the NDC as their basic requirement. Then, from an economic standpoint the most fundamental question is whether the NDC has a socially valuable function, in the absence of which the whole issue of tax breaks would be wiped out, at least normatively. In the present paper we address this question with reference to non-profits that do
not rely on donations.

In donative non-profits the NDC’s role is to solve the conflict of interest between donors and residual claimants by eliminating the latter and thus removing a major obstacle to donations (Hansmann, 1980; Fama and Jensen, 1983; Ellman, 1983). A frequent motivation of giving is the presence of a public-good component in the services produced by the recipient institution. As a matter of fact, the various forms of other-concern behind donations—altruism, civic-mindedness, etc.—often conceal donors’ own consumption of public goods of one sort or other. It is then no surprise that formal analyses mostly focus on non-profits that supply public goods and depend on donations of money or labour for their production (Weisbrod, 1975; Preston, 1989; Bilodeau and Slivinsky, 1996, 1998; Glaeser and Shleifer, 2001; François, 2003; Vlassopoulos, 2009; Gathak and Mueller, 2011). In the theoretical literature altruism plays a role on another plane too—that of management—as these organizations are usually assumed to be run by an altruistic manager/entrepreneur, without the active participation of customers/external beneficiaries in the decision-making process. In other words, the literature mostly deals with entrepreneurial non-profits, as Hansmann (1980) calls them, with an other-concerned management whose incentives are affected by the NDC. This theoretical scheme is applied with few variations also to non-profits that obtain their income from the sale of goods or services such as, most notably, education and healthcare (that some call commercial community-benefit enterprises, cf. Malani and Posner, 2007). The usual justification of the NDC for this class is actually a replica of the traditional one for donative non-profits with customers taking the place of donors and the constraint acting as a harness to controllers’ exploitation of asymmetric or non-contractible information about product quality at the expense of customers (Nelson and Krashinsky, 1973; Nelson, 1977; Hansmann, 1980; Easley and O’Hara, 1983; Brekke et al. 2011, 2012). In this view the NDC’s effects are typically on managers’ choices and, with few exceptions (e.g. Ben-Ner, 1987), formal analyses focus on entrepreneurial non-profits with an essentially self-appointed and autocratic management. This, however, is only part of the story, as in many cases non-profits’ management is subject to an upper layer of control.

An important class where this occurs is that of mutual non-profits, as they are often called after Hansmann (1980), whose income typically comes from sales and where ultimate control rests with customers, that is individuals who have a selfish interest in the firm’s activity, typically as consumers or users. These are either directly run by customers themselves or, if size prevents it, by a professional management that customers appoint and have the power to replace. If the assump-
tion that all decisions are taken by an altruistic and autocratic management is on
the whole acceptable for donative non-profits, it is instead too restrictive for mu-
tual non-profits where, directly or indirectly, customers have a role in running the
firm. As a matter of fact, the entrepreneur of a mutual non-profit is best seen as a
body of two groups interacting with one another in different roles and whose mo-
tivations are a mix of selfish and altruistic ones. Managers and controllers of last
resort—customers in our case—are complementary, not substitute roles, and man-
agerial models of the altruistic entrepreneur, assuming that customers are inactive
as entrepreneurial decision-makers, offer only a partial view. Our chief interest
is in the role of customers as controllers. When this is allowed for, there imme-
 diately arises the question of how the NDC affects their choices, as distinct from
the management's, and in particular the phenomenon of stakeholder exploitation.
In this paper we take a first step towards a fully-fledged theory of the non-profit
(collective) entrepreneur, allowing for an active role of customers in the running
of firms. Like managerial theories of non-profits, we too assume that one of the
two groups of controllers is inactive, but here the focus is on the other side: in our
model it is customers who make decisions and thus are the only ones to play an
entrepreneurial role.

Altruistic motivations are an essential ingredient of managerial donative non-
profits, without which they can hardly exist. By contrast, in mutual non-profits
customer-controllers can and normally will be selfish, either pure or impure, with-
out prejudice for the existence of the organization. The situation we study is
thus opposite to that of donative non-profits: customers are necessarily selfish
and only incidentally altruistic, while the reverse occurs with donors and/or man-
agers. To keep the analysis simple, we lay aside the case of impure selfish cus-
omers by assuming that mutual non-profits sell pure private goods and customers
have no altruistic motivations, so that the collective entrepreneur is not altruistic
either. In other words, the focus is on how the NDC's affects the behavior of
purely selfish entrepreneurs, who are personified by customers in our model—
customer-entrepreneurs, to be precise. In sum, we focus on organizations where
the produced good is purely private—like most public utilities and some personal
services—, donors are absent and control rests with customers. In particular, we
want to check if the claim by existing theories that the NDC is capable to prevent
the exploitation of key stakeholders still holds when it comes to mutual non-profits
where the individuals to be protected are customers, not donors. The main find-
ing is that in this context there are significant cases where the NDC is socially
harmful.

Our analysis has relevant implications not only for tax but also for privatiza-
tion policy. The 2000s were witness to the growth in several countries, notably Europe and South America, of a strong political opposition to traditional privatization involving for-profits. This climate fostered a policy debate on service provision by entities like citizen enterprises, community enterprises, etc.—which are just different names for mutual non-profits—and this solution started to be advocated as a remedy to privatization failures (cf. Mayo and Moore, 2001; Maltby, 2003; Reed and Stanley, 2005, to quote just a few). When the supplied good is a privatized public service, it is also likely, both for political and economic reasons, that customer-citizens are called to take part in the running of these enterprises in a way or other. Then the non-profit type most relevant for privatization policy is likely to be the mutual one. This policy debate, however, lacks firm theoretical foundations and mostly relies on common sense. Bennet et al. (2003) and Bennet and Iossa (2010) make the case for the handover of public services to non-profits on a more theoretical basis but their analysis is still preliminary, as it does not address the role of the NDC and gives no consideration to the firm’s internal organization and governance. A fully-fledged theory of privatization policy should allow for all firm types, including not-for-profit firms like customer-owned and non-profit ones. We take a first step towards such a theory. The main difference between customer-owned and non-profit firms is the NDC, which is present in the latter but not in the former (for a full discussion of this point see section 2.1 below). Our paper’s target is the normative implications of the NDC and, to isolate them, we disregard other possible differences like altruistic attitudes, by developing a firm model with private goods and selfish customers playing an entrepreneurial role.

The paper is organized as follows. In section 3 we show that with homogeneous preferences the NDC is either irrelevant or harmful from an allocative standpoint, which means that its final effect on social welfare is negative owing to welfare losses arising from tax distortions. With heterogeneous consumers a new phenomenon arises—minority exploitation. This is a form of rent-seeking that is realized either by keeping prices too high or too low, with the possible generation of accounting losses (section 4.1). The NDC is shown to work poorly in both cases, for different reasons. When distortions involve losses, the problem is that the NDC is ineffective. On the other hand, even when it is effective, the NDC may not be good, as in some cases it yields outcomes that are socially inferior to those obtained in the presence of minority exploitation (section 4.2). Section 5 deals with non-member customers. It shows that non-members’ exploitation is of the same nature as the minority’s. The presence of non-members, however, has a dampening effect on distortions and makes the NDC more valuable than in their
absence.

2 Set-up

Our analysis of the NDC is based on the comparison of two specific organization types and our first step is to look at the institutional features identifying them.

2.1 Institutional framework

One of the two firm types we study in the present paper is a customer-owned supplier to final consumers of a private good in the economic sense. This organization is similar to a consumer cooperative and in the following will be referred to as cooperative.\(^1\) The other organization type is identical to the former in every respect, except for being subject to the NDC. In Hansmann’s (1987) terminology this is a mutual commercial non-profit,\(^2\) hereafter called non-profit for brevity.\(^3\) The discriminating feature is then the NDC and it is crucial to explain precisely what is meant by this term here.

The main difficulty in dealing with the non-distribution constraint is that it is not specified in the same way everywhere. There is always a core provision, universally recognized by law, prohibiting non-profits from distributing (positive) accounting profits to those who act as controllers in various capacities—members, shareholders, managers, etc.—according to the legal and organizational context. This is usually accompanied by further provisions like restrictions on the kinds of business activity that a non-profit is allowed to undertake (as for example in Italy,  

\(^1\)In this paper the term is used without reference to a specific legal system. However, it may be useful to identify the closest legal forms to our concept. In some countries, notably the USA, there is some overlap between non-profits and cooperatives, while in others the two fields are strictly separated. US cooperatives (mutual-benefit organizations) are prohibited from distributing net earnings in cash, though they are allowed to supply free services to their members (cf. Malani and Posner, 2007), which is just a different mode of surplus distribution. In most European countries—like e.g. Italy—cooperatives are instead exempt from the NDC and can distribute net earnings in cash and other forms. Our “cooperative”—customer-controlled and not subject to the NDC—is then a stylized representation of European consumer cooperatives and this justifies the use of the name (though, it is to be remembered, such use is at variance with the US legal jargon).

\(^2\)Another term in use is “consumer-oriented NFP”, see Malani et al. (2003).

\(^3\)Note that this is just a short-hand denomination for the only kind of non-profit allowed for in the paper—customer-controlled—while, properly speaking, non-profit (firm) is every firm subject to the NDC (note that we use “firm” for non-profits too, unlike some authors who avoid it, e.g. Spulber, 2009).
where the law specifies a list of admissible activities for social enterprises) or the extension of the distribution prohibition to specific groups of non-controllers (as e.g. in the USA, where the distribution ban applies to all affiliated persons and employees of a non-profit, cf. Malani and Posner, 2007). Another category of frequently observed auxiliary provisions concerns the distribution of retained profits. Whenever a company can be sold in the market, owners are able to recoup some of the retained profits by selling their stakes, as the company’s value normally embodies at least part of accumulated profits. For this reason in some countries the NDC includes restrictions on ownership transfers or limitations on the amount of proceeds from a firm’s sale that can be legally appropriated by controllers—a system generally known as “asset lock”. Secondary provisions like this and the others mentioned above actually show considerable variation across countries and we will ignore them. In our analysis we focus on the core of the NDC, that is the ban on all money transfers from firm to controllers—whoever they may be—out of the firm’s current profits.

It is to be noted that the prohibition of money transfers to firm-controllers is of little avail if it is not accompanied by some restraint on perks, as the latter can be artfully used to circumvent it. Suppose that it were possible for firm-controllers to appropriate the full value of the surplus \( \pi \)—i.e. the difference between total revenue and total opportunity cost—through perks.\(^4\) Obviously, in this case the prohibition of money transfers would be wholly ineffective and everybody’s behaviour would be just the same as in its absence.\(^5\) Then, for the NDC to have any effect at all, not only must money transfers be ruled out but there must also be involved some cost in distributing benefits through perks, or, in other words, their value to the recipients must be lower than the amount spent to buy them. The case where the NDC is wholly ineffective is obviously not interesting to us and we will ignore it. But also cases where it is partially effective—i.e. a non-null part of the surplus can actually be transferred to firm-controllers under it—are little relevant for our purposes. Imperfect enforcement is a potential inefficiency cause of its own, as the NDC’s benefits, whatever they may be, are likely to be smaller, the less stringent the constraint is, while its costs in terms of fiscal incentives remain the same. Here we are interested in other types of inefficiency caused by

\(^4\)Embezzlement is another way, but this is illegal. Even perks are sometimes obtained illegally. Illegal behaviours are certainly an issue in non-profits, as documented e.g. by the press news reported in Gibelman and Gelman (2004), but we confine the analysis to legal ones.

\(^5\)Formally, if as in Malani et al. (2003) we denote by \( f(\pi) \leq \pi \) the subjective value of the perks purchased by spending \( \pi \), we have no effectiveness when \( f(\pi) = \pi \) for all \( \pi \), partial effectiveness when \( 0 < f(\pi) < \pi \), and full effectiveness when \( f(\pi) = 0 \) for all \( \pi \).
the NDC, which are best analyzed in pure form by abstracting from potentially reinforcing factors like imperfections in the constraint’s effectiveness. Thus, we do away with enforcement issues and concentrate only on organizations where the NDC, when adopted, is perfectly enforced and no benefits are ever passed in any form to firm-controllers.\textsuperscript{6}

Now let us turn to firm governance. The governance structure of mutual non-profits and customer-owned cooperatives (but the same is true of for-profit corporations, for that matter) typically consists of a two-layer control hierarchy: a “member”/owner layer and a subordinate managerial layer.\textsuperscript{7} In a mutual non-profit the former is necessary, as there cannot be one without customers’ participation in the internal decision-making process (indeed, if a non-profit’s management were wholly independent of its customers, we would have what Hansmann calls an entrepreneurial non-profit, not a mutual one, cf. Hansmann, 1987). By contrast, there can exist mutual non-profits without a proper management or with a management in a purely executive role that just implements decisions taken elsewhere. The presence of a true management, endowed with real decision-making powers, brings with it the well-known problem of managerial agency that affects both for-profits and not-for-profit firms but not in the same way, as has been noted in the literature at least since Jensen and Meckling (1976). Our chief interest here is in the impact of the NDC on customer-controlled non-profits. In view of this we disregard managerial agency issues in non-profits and, in order to allow level-ground comparisons, we do the same with cooperatives as well. In the same vein, we also assume that the latter have the same governance structure as mutual non-profits, so that the presence/absence of the NDC is the only difference between them in our model. Note that cooperatives as we have characterized them are in fact identical to Hart and Moore’s (1996). In sum, we assume that in both organization types all strategic decisions are collectively taken by customers through democratic voting and passed to a management that implements them exactly.\textsuperscript{8}

\textsuperscript{6}However, note that our model can be interpreted in such a way as to allow for perks as well. If, instead of representing their value as a profit loss, we interpret them as a component of the total cost $c(x)$, the present model can actually deal with perks that are not decision objects (i.e. whose value is exogenously given) without formal changes.

\textsuperscript{7}That is, formally subordinate. Of course, a firm’s management may have more actual power than ultimate controllers/owners, and it may act in conflict with the latter’s aims, as the managerial firm theory contends, or in accordance with them, as claimed e.g. by entrepreneurial management theories (cf. Teece, 2016). All this is of no concern to us here.

\textsuperscript{8}This assumption is often made in models of for-profit corporate governance as well, cf. Prinz and Burg (2013) for a recent example.
2.2 The model

We analyze the two stylized organizations outlined in the previous subsection within a very simple economy. There are $n$ consumers of a private good of given quality that is sold in the market at a uniform price $p$ (our non-profits do not draw revenues from donations). The production technology is given and the cost for producing a total quantity of the good $x$ is $C(x) = c(x) + F$, $F \geq 0$. The function $c(x)$ is assumed to have the usual properties $c(x) \geq 0$, $c'(x) > 0$. Note that we do not impose restrictions on the sign of second derivatives but it is useful to assume that the average cost function $AC(x)$ is monotonic—increasing or decreasing—so that the inverse $AC^{-1}(x)$ is defined for every $p > 0$. We also assume that the whole production of the good is carried out by a single firm. This assumption requires some justification.

As we will see, some of the NDC’s most relevant effects are on pricing. Then, for a meaningful analysis one has to allow for firms’ market power. There are sectors, like healthcare and education, that are characterized by monopolistic competition and where non-profits normally operate vis-à-vis other service providers. One question in such contexts is how the presence of non-profits affects market equilibria and allocation (Lakdawalla and Philipson, 2006; Bolton and Mehran, 2007). Our focus, however, is on another aspect—namely, stakeholder welfare and the NDC’s impact on it. Thus, laying aside market structure issues, we concentrate on the baseline case of the monopoly market—the simplest context where market power is present on the supply side. The merit of this choice is that it allows to expose in the most convenient way phenomena that occur also in markets where monopoly power mingles with competition. Then, the present analysis, though not directly dealing with such situations, is relevant to them too, as it is the starting point for any research effort in this direction. Finally, it is to be noted that, though mainly motivated by theoretical reasons, the choice of focusing on monopoly has an empirical basis too, as many real-world non-profits do operate in monopoly or quasi-monopoly markets, especially in the field of personal services.\footnote{In perspective there might be even more non-profits operating in monopoly markets, if more public services were delegated to citizen-controlled non-profits, as some critics of classical privatization advocate (see the Introduction). At any rate, citizen enterprises running public utilities in monopoly are already a reality in various parts of the world, see Spinicci (2014) for a review. Note that even capital-intensive public services are amenable to be supplied by non-profits, as the cases discussed in Bennet et al. (2003) show.}

Each of the $n$ consumers is identified by an individual preference parameter
\[ \theta_i, i = 1, \ldots, n, \] which is harmless to assume non-decreasing in \( i \), i.e. \( \theta_i > (\leq) \theta_j \) only if \( i > (\leq) j \), so that a higher index never corresponds to a lower propensity to consume. Individual preferences are represented by a quasi-linear utility function

\[
u(x_i, \theta_i) + W_i. \tag{1}
\]

Function \( \nu(x_i, \theta_i) \) represents the value of consumption \( x_i \) to consumer \( i \) and is assumed to have the standard properties \( \nu_1(x_i, \theta_i) > 0 \), \( \nu_{11}(x_i, \theta_i) < 0 \) and \( \nu_2(x_i, \theta_i) > 0 \). It is also useful to assume \( \nu_{12}(x_i, \theta_i) > 0 \) (single-crossing property), which allows a considerable simplification of notation and exposition at no cost (nothing substantial in our analysis depends on this assumption). The second addendum of (1), \( W_i \), is the individual net wealth, which is specified differently in the different institutional contexts.

In the standard case, where there is no financial participation of consumers in the firm that supplies them, \( W_i \) is defined simply as \( W_i = M_i - px_i \), where \( M_i \) is \( i \)'s initial money endowment and \( px_i \) her expenditure on the good. By contrast, membership in a cooperative entails money transfers from firm to members or vice versa, whereby net wealth is given by \( W_i = M_i - px_i + \phi_i(x_1, x_2, \ldots, x_n) \), where \( \phi_i(x_1, x_2, \ldots, x_n) \) is the share of net earnings assigned to member \( i \) under some sharing rule. Accordingly, the individual welfare function (1) is specified as follows

\[
u(x_i, \theta_i) - px_i + M_i + \phi_i(x_1, x_2, \ldots, x_n). \tag{2}
\]

In the following analysis we will deal only with situations where the firm supplies exactly what the market demands, i.e. \( x = \sum_i x_i \), and consider only exhaustive profit-sharing rules, i.e. such that

\[
\sum_i \phi_i(x_1, x_2, \ldots, x_n) \equiv px - c(x) - F.
\]

Note that the latter condition rules out accumulation of profits and, by assuming this, we actually choose not to analyze the NDC’s impact on the firm’s reserves. Note moreover that, if there are losses, i.e. \( px - c(x) - F < 0 \), then \( \phi_i(x) < 0 \) for some \( i \).\(^{10}\) A generic member \( i \) of a mutual non-profit has a similarly defined utility

\[
u(x_i, \theta_i) - px_i + M_i + \min \{0, \phi_i(x_1, x_2, \ldots, x_n)\} \tag{3}
\]

where the last addendum embodies the rule that profits are never distributed, while losses are. Individual demands for a given \( p \) and sharing rule \( \phi_i(\cdot) \) are chosen by

\(^{10}\) Another paper that explicitly takes into account the distribution of losses among members (citizens) is Corneo (1995).
consumers so as to maximize their objective functions (2) or (3), according to whether the NDC is or is not in force. Since existence issues fall out of this paper’s scope, we just assume that the objective functions are concave, so that their maximization yields well-defined demand functions.

The last point to be addressed is who the members are in our model. By “member” we mean an individual who has ultimate control over the firm jointly with others. Consumer-owned mutual organizations are those whose members are also consumers of the goods produced by them, but of course not all consumers need to be members and in the real world we actually observe that customers of such firms normally include non-members too. The basic aim of the NDC is to prevent exploitation of some customers by others and this is the main subject of our analysis too. A few papers in the literature have concentrated on the exploitation of non-member customers by members (Kelsey and Milne, 2006, 2008). This is indeed a possibility but it is not the only one and not even the most relevant one. As a matter of fact, the exploitation phenomenon is more complex than represented in these papers and the most important aspect is intra-member exploitation, since rent-extraction from non-members can be shown to be of the same nature as minority member exploitation, of which it is just a variant, as we will see in due course. This justifies a separate analysis of situations with and without non-member customers. For this reason we start by assuming the coincidence of consumers and members (whose number is $n$) and the assumption is maintained until section 5, where we will deal with non-member customers.

3 The benchmark case: homogeneous preferences

The NDC has a different impact on firm behaviour depending on the nature of its owners. Consider the case of owners that are not customers. Provided that prices can be set freely, will the NDC have the firm choose the budget-balancing price? Not necessarily. As we have argued in the previous section, a first reason that may induce to keep prices above this level, despite the NDC, is perks. Another reason for a positive surplus is the aim to accumulate reserves. If both motives are absent, however, prices will indeed be equal to the average opportunity cost. The behaviour of a customer-owned cooperative is different. In such a firm profits/losses can be freely passed to members and their demands for the good depend on both the purchase price and how accounting surpluses—positive or negative—are shared among them. Losses, of course, cannot be sustained indefinitely without being covered by members and in this regard we assume that
they are fully covered within each accounting period. This allows to treat them formally in the same way as profits—a member’s loss share is just the amount of money she is to transfer to the firm by way of loss compensation at the end of the period—and thus simplify the analysis considerably.

Let us see how consumer demands are defined for a generic individual identified by parameter $\theta_i$. In the case of no profit-sharing each individual has a standard demand, denoted $x_i(p)$, that is obtained by the optimization of (2) without the term $\phi_i(\cdot)$ (note that, to simplify the formalism, we avoid to explicitly indicate $\theta_i$ as an argument of the natural demand function $x_i(p)$ and the same is done below with the other demand functions that we will work with). When there is profit-sharing, consumer-members’ demand functions are defined in a slightly more complicated way, since this introduces strategic interactions between them. Suppose that the profit-sharing rule is the uniform (egalitarian) one, i.e.

$$\phi_i(x_1, x_2, \ldots, x_n) = \frac{px - c(x) - F}{n}.$$ 

The demand of a generic individual $i$ now depends on the fellow-members’ demands $x_{-i}$, as they contribute to determine her profit share too. Then, for every $p$ the vector of individual demands under the uniform profit-sharing rule $(\hat{x}_1(p), \ldots, \hat{x}_n(p))$ is a Nash equilibrium and $\hat{x}_i(p)$ are the Nash demands at $p$. In a Nash equilibrium each $\hat{x}_i(p)$ is a solution of

$$\max_{x_i} \left[ u(x_i, \theta_i) + M - px_i + \frac{p(x_i + \hat{x}_{-i}(p)) - C(x_i + \hat{x}_{-i}(p))}{n} \right]$$

and hence there must hold

$$u_1(\hat{x}_i(p), \theta_i) - p + \frac{1}{n} [p - C'(\hat{x}(p))] = 0 \quad (4)$$

(for further details see Lemma 2 of the Appendix). The last addendum is the sharing effect, which causes demand distortions relative to the situation where no profit distribution occurs.$^{11}$ More precisely, when preferences have the usual properties and there exists a unique first-best price $p^\ast$, $p^\ast = C'(x(p^\ast))$, $x(p) \equiv \sum_{i=1}^n x_i(p)$,

$^{11}$Farrell (1985) was perhaps the first to analyze the impact of owners’ consumption motives on firms’ pricing policies. Our analysis differs from Farrell and related literature (Mas-Colell and Silvestre, 1991; Hart and Moore, 1996, and others), as we allow for the sharing effect induced by the dependence of profit shares on own consumption and its strategic implications. In a word, we work with demands $\hat{x}_i(\cdot)$ instead of “natural” demands $x_i(p)$.
the demand \( \hat{x}_i(p) \) is larger than its natural counterpart if \( p > C'(\hat{x}_i) \) and smaller in the reverse case, as implied by condition (4) (cf. Lemma 2 of the Appendix for the details). In other words, the sharing effect distorts demand upward (downward) when it is positive (negative) and the only price where no distortions occur—and \( \hat{x}_i(p) \) coincides with \( x_i(p) \)—is the first-best price itself. Under our assumptions on utility functions the quantity demanded at any \( p \) is strictly increasing in \( \theta_i \), both with and without profit-sharing, as is shown in Lemma 3 of the Appendix.

Now we focus on the case of equal members, \( \theta_i = \theta_j \), for all \( i, j \) (recall that by assumption all customers are members as well). In this case demands are equal across consumer-members irrespective of the sharing rule. It is rather intuitive that, if prices can be set freely, in these circumstances the cooperative will choose the first-best price itself. If a consumer-owner increases her consumption by one unit, she earns the marginal net surplus \( [u'(\hat{x}_h(p)) - p] \) plus an additional income from her participation in profits, equal to \( \frac{1}{n} [p - C'(\hat{x}(p))] \). If the income variation is positive, demand is raised beyond the level that would obtain without profit-sharing, while it is pushed below for negative values. With homogenous preferences, at every Nash equilibrium of the demand game individual welfare is the same for all consumer-owners, that is exactly \( 1/n \) of the social welfare. This means that there is no divergence between individual and social interests, and for this reason members unanimously prefer the price where no distortion occurs—the first-best price—as is formally proved in Lemma 4 of the Appendix. As a consequence, the uniform sharing rule is undominated by any other rule (though it is not uniquely dominant, as there are other rules equivalent to it from a welfare standpoint).

Enter now the NDC. Its impact on individual demands depends on the market price, and particularly on whether this is such as to generate profits or not, as the constraint is binding only when profits arise. Let consumer-member \( i \)'s demand under the NDC be denoted by \( \tilde{x}_i(p) \). It is intuitive that for all prices larger than the second-best price, \( p^{**}, p^{**} = AC(x(p^{**})) \), demands under the NDC must coincide with natural demands, \( x_i(p) \). Indeed, if the market price is set above \( p^{**} \), there arise profits that do not accrue to members, whereby their actual demands cannot be but \( x_i(p) \). More precisely, for all \( p \geq p^{**} \) there holds \( \tilde{x}_i(p) = x_i(p) \) and the vector of natural demands is a Nash equilibrium, as is proved formally in Lemma 5 of the Appendix. Moreover, indirect utility functions under the NDC coincide with ordinary ones, \( u_i(x_i(p)) - px_i(p) \). This fact allows to identify the equilibrium price without difficulty.

There are two main cases to be considered. One is when the first-best price is larger than the second-best one, \( p^* \geq p^{**} \), as in Figure 1 (the graph gives a
representation of this case when there are global diseconomies of scale and $F = 0$). Note that in the figure the aggregate demand curve under the NDC, $\tilde{x}(p) \equiv \sum_{i=1}^{n} \tilde{x}_i(p)$, and the “natural” one, $x(p)$, coincide above $p^{**}$ but differ below it, as indicated by the two arcs—solid and dotted—departing from $p^{**}$. Since individual utilities are decreasing in $p$, any profit-making price will support a lower welfare than $p^{**}$ (note that, differently from an ordinary company, a cooperative cannot be sold and hence there is no stock-value effect here to compensate for surplus losses due to price increases). Then, whenever it is impossible or too costly to distribute the firm’s surplus in any form other than money transfers, no profit-making price will emerge in equilibrium when $p^* \geq p^{**}$. A first fact to be noted, then, is that the pricing policies of the investor-owned and customer-owned firms generally diverge under the NDC. A further point of interest is that the NDC is not always a Pareto-efficient arrangement, since the welfare-maximizing price $p^*$ is attainable without it (e.g. under the uniform sharing rule) but not if the NDC is in force, as is clear from Figure 1. The other case to be considered is $p^* < p^{**}$. This is possible only when there are economies of scale at work. Here the first best is a Nash equilibrium that is unaffected by the NDC, since under it losses can be freely distributed among members. Then, we are sure that the choice will fall on a price below $p^{**}$ ($p^*$ is preferred to any $p \geq p^*$ by customers) and the constraint has no allocative effect. This, however, does not mean that it has no effects at all, but we will come back to the point later on (sec. 4.2).

Note that this is not the same outcome that would obtain under the Cost-of-Service regulatory rule, which, despite the resemblance, differs from the NDC in that it allows for a “normal” profit.
The upshot of the preceding analysis is that under preference homogeneity the NDC is from an allocative standpoint either irrelevant, as in the case of natural monopoly, or outright harmful, as in the diseconomies-of-scale case of Figure 1, where the optimal non-discriminating price entails positive profits and the prohibition to distribute them causes deadweight welfare losses. All this is summarized in the following proposition.

**Proposition 1.** If members have equal preferences, the NDC is never strictly desirable from a social standpoint for mutual non-profits.

It must be stressed that the proposition depends on the assumption that neither dividends nor perks accrue to the firm’s controllers (see sec. 2.1). Of course, if this condition were only partially met (for example owing to a less strict formulation of the constraint or imperfect enforcement), the result—and those of the next sections, for that matter—would no longer hold. It is nonetheless reasonable to expect that in its place there would hold some weaker variant of it but we stop here, as our aim is to develop a full analysis of the pure case, which constitutes the basis for any extension in this direction.

### 4 The social value of the NDC with heterogeneous preferences

An implication of the previous section’s results is that, if the NDC is to be socially desirable, customers must differ in preferences (or wealth, for that matter, though the latter plays no role in the present analysis, owing to the assumption of zero income effect implicit in the quasi-linearity of utility functions). When members have heterogeneous preferences (or different wealth), it is possible that some of them are able to cause social surplus redistribution in their favour, much like a profit-seeking monopolist. According to the conventional wisdom the NDC’s main function would be just to prevent such forms of opportunism.\(^{13}\) This, however, is not always the case and exploitation phenomena are possible under it too.

\(^{13}\)Statements to this effect abound in public policy analyses and advocacy papers. Here are just a few examples: “With the pricing conflict removed, the Consumer Service Corporation [another name for our mutual non-profit] will find its interests fundamentally aligned with those of its users . . . ”, Maltby (2001); “The strongest argument in favour of mutualisation is that monopoly services should be under consumer control because . . . it stops anyone else from exploiting the natural monopoly, and consumers will hardly exploit themselves . . . ”, Birchall (2002). Sensible though they may appear, these claims have no theoretical foundation and there are indeed circumstances where they are false, as we will see.
Moreover, even when it actually succeeds in restraining opportunism, its effects are not always desirable in welfare terms and it may be even better to do without it, as we will see in the next subsections.

4.1 Exploitation of minority members

In member-controlled organizations firm choices are typically collective and are often marred by rent-seeking—a general phenomenon that here takes the specific form of minority exploitation through profit-sharing. In this section we make a detailed analysis of it, and particularly of the specific means and mechanics of minority exploitation in our context.

Consumers are heterogeneous if $\theta_i \neq \theta_j$ for some $i, j$. To avoid unnecessary complication, we focus on the case of $\theta_i$ strictly increasing in $i$, i.e. $\theta_i > (\leq) \theta_j$ iff $i > (\leq) j$. Under consumer participation preference heterogeneity implies that pricing is a collective-choice problem that here we model as a democratic voting process (cf. sec. 2.1). Member $i$’s preferred price maximizes the indirect utility function under the uniform sharing rule

$$U(p, \theta_i) = u(\hat{x}_i(p), \theta_i) - p\hat{x}_i(p) + \frac{p\hat{x}(p) - C(\hat{x}(p))}{n}$$

where $\hat{x}_i(p)$ are the individual Nash-demand functions as defined in section 3 ($\hat{x}(p) \equiv \sum_i \hat{x}_i(p)$). Note that the concavity of objective functions (1) is not enough to warrant that functions (5) are concave too. As is well known from the analysis of standard monopoly, the problem arises because the profit function may be non-concave even when the cost function $C(x)$ is convex and utility functions are concave. On the other hand, the analysis is greatly simplified under the single-peakedness of $U(p, \theta_i)$ in $p$ and for this reason we make this assumption without further inquiring into the conditions that ensure it, as the literature generally does too (for our purposes it is enough to know that there do exist environments, like the linear-quadratic one, where the property holds, cf. Doni and Mori, 2014, Lemma 2.1, for the details). Then the maximizer of (5), $p_i^*$, is the solution of the following (FOC) equation

$$-\hat{x}_i(p) + \frac{1}{n} [p\hat{x}'(p) + \hat{x}(p) - C'(\hat{x}(p))\hat{x}'(p)] = 0$$

$^{14}$Minority exploitation is not confined to the present context at all. Another one where it is relevant is political democracy, as pointed out by the public-choice school (see Buchanan and Tullock, 1962, and later contributions).

$^{15}$Throughout the paper we restrict ourselves to internal optima.
which can also be written as
\[
\left[ \frac{\hat{x}(p)}{n} - \hat{x}_i(p) \right] + \frac{\hat{x}'(p)}{n} \left[ p - C'(\hat{x}(p)) \right] = 0. \tag{7}
\]

The reader will have noted that the latter is nothing but a generalization of the standard equilibrium condition for a monopoly market (indeed, when there is no consumption by member \(i\), the condition reduces to the usual one). A point to be stressed is that there is a fundamental trade-off between the two components of member welfare, consumer surplus and profit share, as surplus is decreasing, while profit is increasing in price). Equation (6) solves this trade-off by requiring that marginal consumer surplus be equal in absolute value to marginal profit share (by Roy’s identity the first addendum is the marginal surplus; the second one is the marginal profit share). Thanks to the single-peakedness of functions \(U(\cdot, \theta_i)\), the median voter theorem is applicable to the collective-choice problem under consideration and thus the median member’s preferred choice, \(p^*_m\), is the equilibrium price (to avoid existence issues, we just assume that a pivotal voter \(m\) always exists). A useful formal simplification is to assume, without loss of generality, that there exists an average individual \(a\) such that \(\hat{x}_a(p) \equiv \hat{x}(p)/n\). Under this assumption and the property that individual demand functions \(\hat{x}_i(p)\) are strictly increasing in \(i\) (Lemma 3 of the Appendix), equation (7) implies a nice result that is stated in the following lemma.

**Lemma 1.** Members with lower propensities to consume than the average member prefer higher prices than the first best, and vice versa, i.e. \(\theta_i \leq (>) \theta_a \Rightarrow p^*_i \geq (\leq) p^*_a\).\(^{16}\)

The most interesting implication of Lemma 1 is that equilibria are efficient if and only if the median and average members coincide. The intuition behind

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\(^{16}\)This result has old antecedents. The relation between allocative efficiency and the median-average divergence has been a research topic for a long time and a number of results similar to that presented here have been elaborated for a variety of contexts. Perhaps the most important literature strand—starting with Bowen (1943)—is that concerning the allocation of public goods in democratic systems. The first investigations into the allocative implications of the democratic process in cooperatives were elaborated in the 80s of the last century for worker cooperatives. Putterman (1981) and Putterman and DiGiorgio (1985) are two of the earliest contributions in this line. With regard to consumer cooperatives one of the first versions of the result was elaborated by Hart and Moore (1996). The context they analyze is close to ours but, differently from what we do here, they disregard the sharing effect on consumer demand and thus fail to distinguish between \(x_i(p)\) and \(\hat{x}_i(p)\).
it is simple enough. Let us start from an equilibrium with homogeneous preferences. In such equilibria each customer’s surplus share is $\frac{1}{n} CS(p^*)$ ($CS(p)$ denotes the total amount of the consumer surplus corresponding to consumer demands $\hat{x}_i(p)$ under the uniform profit-sharing rule).\(^{17}\) Then consider another situation where the total consumer surplus at $p^*$ is the same as in the previous equilibrium, $CS(p^*)$, but preferences are heterogeneous, so that this is now distributed unevenly among members. If the median member’s marginal propensity to consume is reduced, her marginal surplus will fall below the marginal profit share $\frac{1}{n} \pi'(p^*)$. This means that in the new situation it becomes advantageous for her to raise the price above $p^*$ and thus increase her profit share (note that, when $p^* < p^{**}$, profit is negative, i.e. it is a loss, and increasing profits in fact means decreasing losses). Since the median’s choice determines the equilibrium price, this will be distorted upward when the median marginal propensity to consume is lower than average, $\theta_m < \theta_a$, and vice versa in the reverse case. Note that, in order for somebody to prefer a price other than the first best, there is needed a divergence between the percent profit share, $1/n$, and the ratio of own to total marginal consumer surplus, $|CS'_i(p^*)|/|CS'(p^*)|$.\(^{18}\) As is well known, price distortions cause surplus redistribution and this is the case in our context too. Indeed, what takes place here is the same basic phenomenon as in standard monopoly but for two differences: surplus redistribution now occurs within the group of consumers and, secondly, in certain circumstances rent extraction is attained by setting exceedingly low prices. Let us have a look at these facts.

A first mode of rent extraction consists in setting prices above $p^*$ just as in standard monopoly. This requires that the median member has a lower than average propensity to consume ($m < a$). Raising price above $p^*$ has a twofold effect, transforming consumer surplus into profits and reducing total social surplus. The point is that the decrease in the latter is not distributed evenly among members, owing to their different propensities to consume. To see how, consider for example the case $m < a$, where, as we know, $p^*_m > p^*$ (Lemma 1). On switching from $p^*$ to $p^*_m$, the median member obviously gets an increase in her utility. It is intuitive that this must be true for all of the majority’s members too, as under the median voter theorem (warranted here by the strict concavity of $U(p, \theta_i)$ and the decreasing monotonicity of $p^*_i$ in $i$) what is good for the median voter is generally good for the majority as well. This is indeed the case, as Lemma 8 of the Appendix

\(^{17}\)Note that this is different from the standard consumer surplus arising from natural demands $x_i(p)$ in the absence of profit sharing.

\(^{18}\)If for $i$ there holds $|CS'_i(p^*)| = \pi'(p^*)/n$, then there necessarily holds $|CS'_i(p^*)|/|CS'(p^*)| = 1/n$, since by definition $|CS'(p^*)| = \pi'(p^*)$. 

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formally proves. More precisely, for those who consume less than the median, 
\( i < m \), there hold 
\( \Delta U_i = \Delta CS_i + \Delta \pi/n > 0, \Delta CS_i < 0, \Delta \pi/n > 0 \), and hence 
\( \Delta \pi/n|\Delta CS_i| > 1 \), while for \( i > m \) these inequalities hold with opposite 
sign. This means that for each unit of lost consumer surplus members \( i \leq m \) in-
crease their profit share by more than one unit, while the reverse is true for \( i > m \). Then, raising prices above \( p^* \) causes an increase in profits that is shared equally 
among consumers but whose cost in terms of consumer surplus loss falls mainly 
on the minority (and not evenly among its members either: those who consume 
more bear more of it). In other words, the additional profits generated by high 
prices are realized at the expense of the minority’s consumer surplus. Note that 
it is the whole majority to gain at the expense of the minority and, viceversa, the 
whole minority to yield surplus to the majority. Members of each group, though 
unequal among themselves, thus have a common interest against the other and the 
two groups are in conflict with one another. It is then fully justified to qualify the 
phenomenon as exploitation of the minority by the majority.\(^{19}\)

An interesting point, so far unnoticed in the literature, is that the majority can 
cause rent redistribution not only through high prices but also through excessively 
low ones. This second exploitation mode is even more important for our purposes. 
The logic is actually the same as in the previous case. When \( p^*_m < p^* \), the me-
dian member finds it advantageous to reduce price below \( p^* \) in order to transform 
profits into consumer surplus. Similarly to the previous case, the ensuing increase 
in total consumer surplus is distributed unevenly and those who consume more 
find themselves enjoying a larger increase in consumer surplus for every unit of 
lost profits, as \( n\Delta CS_i/|\Delta \pi| > 1 \).\(^{20}\) In other words, the majority’s surplus is in-
creased at the expense of profits and profit reductions are mainly borne by those 
who consume less (the minority, when \( m > a \))—a phenomenon that never occurs 
in standard monopoly but is nonetheless a form of exploitation. In sum, price 
changes in either direction cause variations in total profits and this is the channel 
by which surplus flows between majority and minority can take place. Manip-
ulation of these flows by the majority through price changes is thus an essential 
ingredient of the suboptimal transformation of consumer surplus into profits, or 
viceversa.

A few remarks are in order. First, in the previous analysis no mention is made, 
as the reader will have noted, of the algebraic sign of profits and indeed everything

\(^{19}\)Redistribution phenomena have long been investigated in the literature—at least since Sen 
(1966)—with regard to worker cooperatives but not to consumer-controlled organizations, with 
which we are concerned here.

\(^{20}\)\( \Delta U_i = \Delta CS_i + \Delta \pi/n > 0, \Delta CS_i > 0, \Delta \pi < 0, i > m > a \).
we have said holds irrespective of it. For any $p$ the profit sign is the same as that of $(p - \hat{p}^{**})$ and, when negative, what we have is a loss, in which case “profit increase” actually means loss reduction.\footnote{$\hat{p}^{**}$ is the break-even price under the uniform sharing rule, cf. section 4.2.} If $p_m^* < \hat{p}^{**}$, there arise accounting losses and this means that the minority actually pays for part of the purchases made by the majority (loss shares, we recall, are nothing but deferred payments in the present context) but nothing else changes in the phenomena described above.

Second, when $p_m^* > p^*$ (which occurs when $m < a$), the median member’s desire is to substitute more consumer surplus for profit than is optimal. In the opposite case $p_m^* < p^*$ (i.e. $m > a$), the direction of change is reversed and the median member prefers to trade off more profit for consumer surplus than is optimal. In other words, while in the former case rent extraction from the minority occurs through the realization of extra profits (or excessively low losses, if profits are negative), in the latter it occurs through excessively low profits (or extra losses).

Lastly, note that the minority’s exploitation by the majority may be socially worse than the exploitation of the whole group of consumers by a classic monopolist. We do not make a full analysis of this point here but an intuitive grasp of it can be quickly achieved. Consider for example the case of diseconomies of scale. In these circumstances, when $p_m^* > p^*$, the textbook monopoly price $p_M$ lies farther from the first best than $p_m^*$, as $p_M > p_m^*$, and the associated welfare loss is necessarily larger, which means that the for-profit firm is always inferior to the cooperative from a social welfare standpoint. By contrast, in the reverse case, $p_m^* < p^*$, monopolistic price may happen to be close to the first best and thus entail a small welfare loss, while the cooperative price entails a larger one.

4.2 Non-distribution constraint to the rescue?

Differently from the standard case, where a monopolist can take advantage of her monopoly power only by making extra profits, in our context exploitation can also take place by generating losses that are laden on the minority. This is why the NDC is not always capable to eliminate intra-member rent extraction. The NDC exhibits the full range of its disfunctions when there are diseconomies of scale, at least locally. For this reason in most of this section we concentrate on convex cost functions, $c''(x) > 0$. Once the analysis of this case is fully spelled out, the effects of the NDC under increasing returns to scale can be quickly dealt with as an extension, which we will do at the end of the section.

The study of Nash demands under the NDC is now complicated by the fact
that indirect utility functions are not differentiable everywhere. There are two benchmark prices that play a critical role in the analysis, the second-best price \( p^{**} \), \( p^{**} = AC(x(p^{**})) \), and the break-even price under the profit-sharing effect \( \hat{p}^{**} \), \( \hat{p}^{**} = AC(\hat{x}(p^{**})) \), \( \hat{p}^{**} < p^{**} \). Above the former and below the latter the identification of Nash demands is a relatively easy task. In the absence of perks it is obviously not advantageous for any member to have prices higher than the second best, \( p^{**} \) (or, if perks are admissible, higher than the price corresponding to the zero level of profits net of perks). More precisely, individual constrained demands under the NDC coincide with \( \tilde{x}_i(p) = x_i(p) \) for all \( i \) and \( p \geq \hat{p}^{**} \), as is proved in Lemma 5 of the Appendix. As a consequence, the constrained indirect utility is decreasing (the higher the price, the smaller the consumer surplus) and no customer will ever prefer a price above \( p^{**} \). Individual demands for \( p \leq \hat{p}^{**} \) are straightforward to determine too. Since below \( \hat{p}^{**} \)—the break-even price with sharing effect—there arise losses whose distribution is not blocked by the NDC, individual demands under the NDC coincide with the unconstrained demands with profit sharing, i.e. \( \hat{x}_i(p) = \hat{x}_i(p) \) (see section 3 for the definition of \( \hat{x}_i(p) \)). All this is summarized in the following proposition.

**Proposition 2.** With diseconomies of scale individual \( i \)'s demand function under the NDC, \( \tilde{x}_i(p) \), coincides with the natural demand function \( x_i(p) \) on interval \([p^{**}, \infty)\) and with the constrained demand function \( \hat{x}_i(p) \) on \([0, \hat{p}^{**}]\), for all \( i \).

The proposition states that under the NDC demand functions are defined piecewise on a partition of the positive real line into three intervals and identifies them on \([0, \hat{p}^{**}]\) and on \([p^{**}, \infty)\). In the remaining interval \([\hat{p}^{**}, p^{**}]\) identification is complicated by the following fact. At prices \( p \) such that \( \hat{p}^{**} < p < p^{**} \) the natural (total) demand \( x(p) \) generates losses, which can be distributed under the NDC, while the unconstrained demand with sharing effect \( \hat{x}(p) \) generates profits, which cannot (recall that with economies of scale there arise profits with the natural demand \( x(p) \) and losses with \( \hat{x}(p) \)). For these prices Nash demands do exist, as we prove in Lemma 6 of the Appendix, but there exist multiple Nash equilibria. Thus, the proof of equilibrium existence is not enough—and further enquiries would be needed—to identify individual demand functions precisely.  

Nonetheless, we can identify aggregate ones in the following way. At prices above \( p^{**} \) both demands with and without profit-sharing effects generate profits, while

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22 The issue of multiple equilibria can be addressed from different angles. At the stage of consumption, when a price has already been chosen, the problem is one of equilibrium selection. A member, however, is not only a consumer but also a legislator. In the constitutional phase members design the cooperative’s rules to be embodied in its by-laws. Since it is in the collective interest to
below $p^{**}$ both generate losses. In between, instead, one generates profits and the other losses. Suppose that at a given $p$, $\hat{p}^{**} < p < p^{**}$, demands are the natural ones, $x_i(p)$, and that in their correspondence there arises a loss (as is the case with diseconomies of scale). The NDC is not binding in this case and losses are shared among members, which makes them revise their demands downward to a level where a profit arises, which instead cannot be shared under the NDC and makes members revise demands again, as if there were no sharing effect. It is intuitive that the only case where this cyclic behaviour is prevented is when the aggregate demand generates neither losses nor profits, i.e. $x(p) = AC^{-1}(p)$ for all $p$ such that $\hat{p}^{**} < p < p^{**}$, as is formally proved in Lemma 6 of the Appendix. The aggregate constrained demand function has thus the shape of the bold broken line in Figure 2.

So far we have focused on the demands arising in the second-stage game where members choose their consumption in response to the price set at the first stage. We now turn to first-stage choices. The equilibrium price is the median member’s preferred price and, to identify it, we must know how it affects individual welfare under the NDC, i.e. her constrained indirect utility function. Let us see how this compares to the unconstrained one, as given by equation (5). Figure 3 represents the constrained and unconstrained indirect utility functions of the median member

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regulate individual demands at prices for which aggregate natural demand exceeds the supply the cooperative wants to implement, it is not unreasonable that at this stage members democratically choose some rationing scheme that eliminates multiple equilibria. However, the identification of individual demands at these prices is not necessary for our argument and we will not pursue the subject any further.
in one specific case, namely when her unconstrained preferred price is larger than the second best, \( p_m^* > p^{**} \). The dotted arc represents the unconstrained indirect utility with profit sharing, \( U_m(p) = u(\hat{x}_m(p), \theta_m) \) for \( p \geq p^{**} \). The solid curve instead represents the constrained indirect utility function under the NDC and consists of three different pieces. At the right of \( p^{**} \) it coincides with the natural indirect utility, as represented by the smooth arc, and at the left of \( \hat{p}^{**} \) it coincides with the unconstrained demand under profit-sharing. The above results (existence of Nash equilibria and characterization of aggregate Nash demands) are instead not enough to identify the slope properties of the function between \( \hat{p}^{**} \) and \( p^{**} \) and for a more precise characterization of indirect utilities there would be needed an analysis of equilibrium selection. However, the exact shape of the curve on this interval is irrelevant for our argument and we will not pursue the subject further (the zigzagged contour indicates that the curve’s shape is undetermined in the concerned tract).

If price distortions are accompanied by positive profits, the NDC is effective (though not necessarily optimal, as we will see). Whenever a company cannot be sold and surplus cannot be distributed through perks, it is obviously not advantageous for the median voter \( m \) to vote for her unconstrained preferred price \( p_m^* \), as this would generate profits that would be eventually retained by the firm. In other words, when \( p_m^* > p^{**} \), the NDC is binding for the median consumer and her choice will be \( p^{**} \) or a lower price (higher prices entail lower utility, cf. Figure 3). If instead \( p_m^* < \hat{p}^{**} \), the median voter’s preferred price generates a loss. This means that the majority can exploit the minority by keeping price low and thus
enjoy an extra consumer surplus whose cost chiefly falls on the latter. This, as we have seen in section 4.1, is a way of extracting rent that is not substantially different from reaping profits at the minority’s expense through high prices, except that this kind of opportunism is not blocked by the NDC. All this is summarized in the following proposition.

**Proposition 3.** With diseconomies of scale the equilibrium price under the NDC is equal or lower to $p^{**}$, if $p^{*}_m > p^{**}$, and equal to $p^{m}_m$, if $p^{*}_m < \hat{p}^{**}$.

The proposition states that, when the median member has a lower willingness to pay than the average member’s, i.e. $\theta_m < \theta_a (p^{*}_m > p^{**})$, the NDC affects the behaviour of the majority, while it has no effects at all if $p^{*}_m < \hat{p}^{**}$ (in the intermediate case, $p^{**} < p^{*}_m < p^{**}$, nothing definite can be said). Then, there are cases where the NDC is certainly effective and others where it is certainly ineffective in conditioning the majority’s behaviour. An important point to be noted is that, when it is effective, there is no guarantee that the NDC eliminates or even reduces exploitation. As we have seen in section 4.1, when the majority push prices below the first best, their aim is to increase consumer surplus at the expense of the minority’s profit share. Therefore, when the majority’s preferred price $p^{*}_m$ is strictly comprised between $p^{**}$ and $p^{*}$, the NDC is indeed effective in forcing the choice away from $p^{*}_m$, but the direction of change is not the correct one, since prices are pushed further down, thus exacerbating exploitation.

In other instances, instead, the NDC does eliminate exploitation but this has an adverse impact on social welfare. When $p^{*}_m > p^{*}$, we know that the majority’s aim is to extract rent from the minority by realizing extra profits at the expense of the latter’s consumer surplus. Of course, if profits cannot be distributed, this kind of exploitation cannot be performed but the outcome may be worse than without it, as can be seen easily. If the NDC is effective, by Proposition 3 the equilibrium price is equal to the second best or lower. Assume for simplicity that the equilibrium price is just $p^{**}$ and moreover that the standard monopoly price, $p_M$, is preferred from a social welfare standpoint to the second best with decreasing returns to scale (note that this is possible only in this case). Now, if the median voter’s unconstrained choice is close to $p_M$, the majority will actually vote for a price close to the profit-maximizing one. In the extreme case where the majority have zero willingness to pay for the good (i.e. they are not directly interested in it), they clearly find themselves in the same position as pure investors, whose utility comes from dividends only, and will act accordingly by choosing just $p_M$, if there are no restrictions on profit distribution.
Figure 4 – A case of Pareto-inefficient NDC with heterogeneous preferences

Figure 4 provides a simplified representation of this case, as the graph ignores the divergence between demands under the profit-sharing effect and natural demands, which is inessential to the present argument (in other words, it assumes that $\tilde{x}(p) \equiv x(p)$). Given a utilitarian welfare aggregation criterion, if the plane portions $\gamma$ and $\eta$ are such that $\gamma > \eta$, the social surplus (the sum of all members’ total surpluses, i.e. consumer surplus plus dividends) is larger at the monopoly price $p_M$ than at the second-best one, and hence the pure monopoly solution without the NDC is socially preferable to the second-best price under the NDC. This simply means that, from a normative standpoint, allowing some exploitation is in this case better than removing it through the NDC. If such an outcome is possible in the extreme case where the cooperative’s majority behave as a pure monopolist, it will be all the more so in those cases where they consume the good supplied by the cooperative as well. When the majority’s willingness to pay for the good is positive but close to zero, the price will be set below the monopoly equilibrium level but close to it. If it is close enough (the chosen price will be closer to it, the flatter the demand curves and the steeper the cost curves), by continuity there will hold the same conclusion, i.e. giving up NDC and having some exploitation will be better than the opposite.

Summing up, the previous analysis has brought to light three disfunctions of the NDC: i) in some cases the NDC does remove all exploitation but this is not socially desirable, as the social surplus turns out smaller than would be if some exploitation were allowed by removing it; ii) in other cases, though effective in conditioning the majority’s choices, the NDC works in the wrong direction and,
instead of removing exploitation, it strengthens it; iii) in a third class of cases
the NDC is simply incapable to affect the majority’s choices, typically when they
entail exceedingly low prices.

An important point is that, when disfunction iii) occurs, the NDC is not neutral
from the standpoint of social welfare. It is true that, in the absence of allocative
effects, there are no direct welfare losses within the market. However, there are
other welfare costs to be accounted for. The non-distribution constraint is uni-
versally sustained by fiscal incentives—tax breaks or subsidies—which have a
social cost of their own. Of course, to calculate the constraint’s final impact on
social welfare, this cost is to be subtracted from the variations in social surplus
induced by it. Then, whenever there is no such variation because the NDC has
no impact on the surplus produced in the market, the final effect on social welfare
will actually be negative and in this case all fiscal support to the NDC should be
withdrawn.

So far the analysis has concentrated on the case of diseconomies of scale.
Under economies of scale some of the above disfunctions cease to arise. Note that
with globally decreasing average costs the first-best price always lies below the
second-best one and the break-even price under the sharing effect is comprised
in between, i.e. $p^* < \hat{p}^{**} < p^{**}$. From this there follow a few nice facts. First,
disfunction i) is no longer possible, as the second-best price is always socially
preferable to the monopoly price. The same is true of disfunction ii), as the NDC,
by pushing the equilibrium price closer to $p^{**}$ from above, actually pushes it closer
to the first best and hence produces welfare gains. What is instead possible is to
extract rent by running an accounting deficit whose burden is borne mainly by the
minority (namely, when this consumes less than the majority), a kind of distortion
(iii) towards which the NDC is wholly ineffective. Ineffectiveness, however, does
not mean neutrality, as we have just argued, and hence mutual non-profits should
not be granted fiscal incentives subject to the adoption of the NDC in industries
with increasing returns to scale.

In conclusion, we have found that with mutual non-profits the prohibition to
distribute profits, though sometimes capable to prevent the majority from exploi-
ting the minority, and thus to enhance social surplus, is not always so and in the
face of a few cases where it is socially desirable there are several others where it
is not. On the whole, when the motivation of donor protection is missing, poli-
cies favouring the NDC through fiscal incentives must be regarded with suspicion.
Commercial mutual non-profits fall into this class and, given the variability of the
NDC’s effects on them, there is no justification for an unconditional fiscal support
to firms complying with the rule.
5 Non-members

So far we have focused on the case where all customers are members too. Of course, this is a simplification. Take for instance consumer cooperatives, with which we are comparing mutual non-profits: they are usually open to non-member patronage and quite often a non-negligible percentage of their customers are not members (internal rules forbidding trade with non-members are rare nowadays, though in the 19th century they were not uncommon). The same is true of mutual non-profits, where customers do not always have a role as controllers ("members"). The presence of non-members, therefore, cannot be ignored. The theoretical literature, on the other hand, has paid little attention to this topic. If a mutual firm is allowed (by the law, internal by-laws, customs) to practice price discrimination, members will obviously choose a different price for non-members so as to extract a rent from them, much like a monopolist serving two separate markets (cf. Kelsey and Milne, 2006). The discrimination case, however, is actually not very interesting from a theoretical standpoint, as it is just a replica of the standard discriminating monopoly. It is not too relevant empirically either. In mutual institutions—e.g. consumer cooperatives, that are the most common type—price discrimination on the basis of membership is not frequently observed and moreover, whenever the supplied good qualifies as a public service, it is likely that price discrimination among users is forbidden by regulatory rules.\footnote{The universal service obligation—to which public services are usually subject—is obviously inconsistent with discrimination based on membership.} In the present section, then, we maintain the assumption of price non-discrimination that we have made before.

The main point is that price non-discrimination by itself is not enough to prevent the exploitation of non-members. If a mutual firm enjoys monopoly power towards non-member customers, it will show a behaviour similar to that of a for-profit monopolist, as there is an obvious incentive for members to set price above the level they would choose in the absence of non-member customers, provided there is no constraint to profit distribution. It is also clear that, under price non-discrimination, neither price nor deadweight welfare losses will be so high as in a for-profit monopoly, because exploiters are consumers themselves. One then expects that in this case there is a definite welfare ranking of the pricing policies of for-profit, cooperative and non-profit firms, with the first being dominated by the second and the second by the third. All this is intuitive enough (see Kelsey and Milne, 2006, 2008, for a formal analysis). A limit of the existing literature is that it
focuses on the exploitation of non-members by members and ignores exploitation within the group of members. As a consequence, it misses not only that the two phenomena affect one another, but also that non-members’ exploitation is nothing but a variant of the former. Here we take a more comprehensive approach. The interesting point is that the analysis of the previous section is largely applicable to this case too, since, as we will see, the presence of non-member patrons has only minor effects which call for small changes to the basic model analyzed above.

A first fact to be noted is that non-member customers, though different from members, are less distant from them than might appear at first sight. One difference is obviously that non-members lack formal control powers but in this respect they are similar to minority members, who lack actual control powers either. There is, however, a basic difference between them: unlike minority members, non-members are excluded from the distribution of profits. In fact, adding non-member patrons to our collective-choice set-up just amounts to an enlargement of the minority without effects on the voting outcome. In other words, the number of individuals that are potential exploitation targets gets larger without threatening the majority’s electoral primacy. Differences in the concerned parties’ behaviours, if any, must then be due to the lack of sharing effects for this class of patrons. Let us look into this more closely.

To deal with non-members there are needed only minor adjustments to the basic model. Suppose that there are \((N – n)\) non-member customers, where \(N\) is the total number of individuals in the market and \(n\) is, as before, the number of members, \(n < N\). In the new situation a member’s indirect utility function at \(p\) is an immediate extension of (5):

\[
U(p, \theta_i) = u(\hat{x}_i(p), \theta_i) - p\hat{x}_i(p) + \frac{p(\hat{x}(p) + y(p)) - C(\hat{x}(p) + y(p))}{n}
\]

where \(y(p) \equiv \sum_{j=n+1}^{N} x_j(p)\) and \(x_j(p)\) is outsider \(j\)’s natural demand function, \(j = n + 1, \ldots, N\) (the other symbols have the same meaning as before). By replacing (5) with (8) we obtain an extended model of the collective-choice process which is not substantially different from the previous one and the results are also qualitatively the same as before. In particular, Lemma 1 holds in the new circumstances too, with the only caution that the average demand is now calculated on a customer population that includes non-members too (formally, \(n\) and \(\hat{x}(p)\) are respectively replaced by \(N\) and \(\hat{x}(p) + y(p)\) in the proof). There are however a few differences to be pointed out.

A first difference concerns member homogeneity. We have seen that in the basic case member homogeneity makes the NDC either harmful or irrelevant from a
social surplus standpoint. This is no longer true when there are non-member patrons, as non-member patronage is a cause of heterogeneity on its own, irrespective of whether members are homogeneous or not. If prices are undifferentiated for all customers, it may indeed be the case that a perfectly homogeneous group of members find it advantageous to push up prices from the first best in order to elicit extra profits from outsiders, which are eventually appropriated by them alone. In such a case prices are distorted upward to cause an increase in total profits and a redistribution in favour of the members who take the decision (insiders get 100% of the variation in profits following the price change, while they bear less than 100% of the variation in total expenditure). We are clearly in the face of the same phenomena as previously described for the case of member-customers only, with outsiders here taking the place of the minority. Differences here are only quantitative and concern the intensity of the effects. In the basic case the minority get a share of profit increases as well, whereas outsiders do not. Thus the marginal profit share is larger and the incentive for the pivotal decision-makers to distort prices is larger if the exploited group is made up of outsiders instead of minority members, other things equal. No exploitation of this kind can instead occur under the NDC. Then, even with perfectly homogeneous members there may occur rent-extraction phenomena that can be taken care of by the NDC. Note that rent extraction from outsiders can only come about by making extra profits, not extra losses.\footnote{Note also that, when members are homogeneous, there never occur downward price distortions. These make sense only if an increase in consumer surplus is accompanied by a profit decrease (loss increase) that affects pivotal decision-makers less than minority ones. This is possible, however, only when the minority is made up of members, as outsiders do not participate in the sharing of profits/losses.}

A second point to be made is that the presence of outsiders has effects in the presence of heterogeneous members too. Imagine to have a situation where there are no outsiders and the majority exploits the minority through high prices. Then imagine that at some point a few outsiders are added to the original customer group: the exploitation effect would obviously be reinforced, as the marginal profit would be pushed up by an increase in marginal revenue, other things equal. Conversely, if in the absence of outsiders the majority pursued a policy of exceedingly low prices, the incentive to do so would be dampened by outsiders’ entry into the market, as the latter would enjoy extra consumer surplus without bearing any of the attendant profit cuts (or loss increases). In this case it may even happen that a price $p^*_m > p^*$ is chosen if a suitable amount of profits can be earned at the expense of outsiders, i.e. such as to outweigh the exploitation that could be
achieved by extra losses borne for the most part by the minority (outsiders instead can never be exploited in this way). The final outcome is uncertain, as it will depend on the relative weight of rent extraction from the minority through extra losses as compared to rent extraction from outsiders’ through extra profits (which in turn depends on the size of each group and their propensities to consume).

To sum up, with non-member customers there can still occur the two exploitation phenomena that occur without, i.e. through excessive prices (extra profits) and through exceedingly low prices (“extra losses”). Apart from the composition of the exploited groups in the two cases (in the latter they comprise only minority members) and their intensity, the phenomena are however the same and the conclusions about the NDC are qualitatively the same too: it may be effective or not, beneficial or not, just as in the basic case and for essentially the same reasons. In the case where it is effective the (upward) price distortions it cures tend to be larger with non-member customers and, conversely, when it is ineffective, (downward) price distortions are naturally mitigated by the self-discipline imposed on the majority by outsiders. In other words, when the NDC succeeds, it does in cases where it is more useful, while, when it fails, it does in cases where it is less needed. Then, on the whole the presence of non-members makes the NDC actually more valuable than without.

6 Concluding remarks

The profit non-distribution constraint is usually viewed as an institutional arrangement aimed to protect donors from firm controllers’ exploitation (and implicitly to promote donations). Donative non-profits, however, are not the only type of third-sector institution and there are indeed many situations where the chief stakeholders are customers, not donors. The basic question we have addressed in this paper is how customer control accords with the profit non-distribution constraint. If this does a good job in donative non-profits, we have seen that with mutual non-profits, which have no donors to be protected, the perspective changes somewhat. The main issue here is the exploitation of some customers by others, rather than the exploitation of donors by managers or owners. We have shown that, when ultimate control rests with customers, the NDC is not always socially desirable, because in some cases it is unable to block this kind of exploitation, while in others it blocks beneficial profit redistribution among members. As a matter of fact, the NDC is sometimes too much, sometimes too little, and overall it appears inadequate as an instrument for curbing exploitation phenomena in customer-controlled
organizations. The conclusion is that, differently from donative non-profits, the appropriateness of tax relief to NDC-adopting institutions is dubious and its universal extension to mutual non-profits unwarranted. The problem, in a word, is the unqualified linkage of tax incentives to the NDC’s adoption. The analysis suggests that the granting of tax incentives should be not be based only on the compliance with the NDC but should take into account the institution’s governance structure as well. As a consequence, a selective application of fiscal incentives appears the most appropriate approach for this class of third-sector institutions.

Appendix

Lemma 2. Nash demand functions under the uniform sharing rule, \( \hat{x}_i(p) \), have the following property

\[
\hat{x}_i(p) \geq (\prec) x_i(p) \iff p \geq (\prec) p^*, \forall p, i. 
\]

Proof. Under the uniform sharing rule, the objective function (2) is specified with \( n \) members as

\[
u(x_i, \theta_i) + M - px_i + \frac{px - C(x)}{n},
\]

where \( x = \sum_i x_i \). Recall that by assumption (sec. 2.2) this function is strictly concave in \( x_i \) (note that, since we do not rule out the case \( C'' < 0 \), in our context the concavity of \( u(x_i, \theta_i) \) is not enough to ensure the concavity of (9) and we need an \textit{ad hoc} assumption). Profit-sharing among members creates a strategic interaction between individual consumption choices and members actually play a game whose strategic variables are the individual demands at the given price. Under the uniform sharing rule, \( i \)'s demand for price \( p \) and \( (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \)
is a maximizer of (9) with respect to $x_i$. Nash demands $\hat{x}_i(p)$ are defined by the following inequalities:

$$u(\hat{x}_i(p), \theta_i) - p \hat{x}_i(p) + \frac{p(\hat{x}_i(p) + \hat{x}_{-i}(p)) - C'(\hat{x}_i(p) + \hat{x}_{-i}(p))}{n} \geq 0$$

(10)

$$u(x_i, \theta_i) - px_i + \frac{p(x_i + \hat{x}_{-i}(p)) - C(x_i + \hat{x}_{-i}(p))}{n}, \forall x_i, \forall i.$$

Note that demand functions $\hat{x}_i(\cdot)$ depend on the number of members $n$ but, since it is a fixed parameter here, we do not make it formally explicit (they also depend on $\theta_i$ but to simplify we avoid indicating it as an explicit argument).

From the comparison of equation (4) with the marginal condition for standard demands without sharing effect, we get that, when $p > (\leq) C'(\hat{x}(p))$, there holds

$$u_1(\hat{x}_i(p), \theta_i) - p < (\geq) 0 = u_1(x_i(p), \theta_i) - p,$$

from which in turn there follows $\hat{x}_i(p) > (\leq) x_i(p)$, by the strict concavity of $u(x_i, \theta_i)$ in $x_i$. Then, the only price where the two demand curves cross is $p^*$ s.t. $p^* = C'(\hat{x}(p^*))$. $\square$

**Lemma 3.** Under any linear profit-sharing rule, $\phi_i(x_1, \ldots, x_n) = s_i \pi, 0 \leq s_i \leq 1, \sum_i s_i = 1$, Nash demand functions $\hat{z}_i(p)$ are strictly increasing in the propensity to consume $\theta_i$.

**Proof.** Let $(s_1, \ldots, s_n)$ be a generic sharing rule, $0 \leq s_i \leq 1, \sum_i s_i = 1$. Nash demands under it at price $p$, $z_i(p)$, are defined in an analogous way to $\hat{x}_i(p)$ in Lemma 2 (see in particular condition (10)). Note that $z_i(p)$ depend on the parameter $\theta_i$ too, though this is not indicated as an explicit argument in compliance with the notational convention adopted in the paper for demand functions. Nash demands (internal equilibria) must satisfy the first-order conditions for the objective function (2), specified for the sharing rule we are considering, i.e.

$$u_1(z_i(p), \theta_i) - p + s_i [p - C'(z_i(p)) + z_{-i}(p)] = 0.$$

From this, by the implicit function theorem, there follows the equality part of the following

$$\frac{\partial z_i(p)}{\partial \theta_i} = -\frac{u_{12}(z_i(p), \theta_i)}{u_{11}(z_i(p), \theta_i) - s_i C''(z_i(p))} > 0.$$  

(11)

The inequality part is instead an immediate implication of the single-crossing assumption (positive numerator) and the strict concavity of the objective function in consumption, $x_i$ (negative denominator). $\square$
Remark. The monotonicity property holds for standard demand functions $x_i(p)$ too, as can be easily checked by replacing $z_i(p)$ with $x_i(p)$ and deleting $s_iC''(z(p))$ in (11). It also holds for functions $\hat{x}_i(p)$, as this is just a special case of $z_i(p)$ ($s_i = 1/n, \forall i$, implies $z_i(p) = \hat{x}_i(p), \forall p, i$).

Lemma 4. With homogeneous members who have quasi-linear preferences and in the absence of non-member customers the uniform sharing rule supports the first best.

Proof. The pricing game has the usual two-stage structure for problems of this kind (see e.g. Farrell, 1985). At the first stage members vote on price; at the second one they define their demands taking the price determined at the previous stage as given. Individual demands have the properties stated in Lemma 2 above.

Let us now calculate the price that would be chosen by a generic member $i$ if she were given the power to choose it (or were to cast a ballot) with homogeneous preferences, i.e. $u(x_i, \theta_i) = u(x_i, \theta_j), \forall i, j$. With a slight abuse of notation we denote the utility function representing the common preferences by $u(x_i)$. It is immediate to see that under our assumptions the Nash demands are unique and equal across members, $\hat{x}_i(p) = d(p), \forall i, \forall p$. Assume to the contrary that $\hat{x}_i(p) \neq \hat{x}_j(p)$ for some $i, j$. Then, owing to strict concavity, the FOC

$$u'(\hat{x}_h(p)) - \frac{n-1}{n}p - \frac{C'(\hat{x}(p))}{n} = 0$$

cannot hold for $h = i, j$ simultaneously, which contradicts the assumption that $\hat{x}_i(p)$ and $\hat{x}_j(p)$ are Nash demands at $p$. The strict monotonicity of $u(x_i), C(x)$ also rules out multiple Nash demand functions, $d^a(p), d^b(p), \ldots$. Since the equality of preferences is common knowledge, everyone knows that, once the price is set, each member will express the same individual demand $d(p) = \hat{x}(p)/n$. Then, the indirect utility function can be written as

$$U(p) \equiv u(d(p)) + M - pd(p) + \frac{pnd(p) - C(nd(p))}{n}.$$ 

The individual voter’s choice is a solution to the maximization problem $\max_p U(p)$, whose first-order condition

$$u'(d(p)) = C'(nd(p)),$$

coincides with the first-best condition (and the first best is $p^*$ such that $u'(d(p^*)) = C'(nd(p^*))$). 

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Lemma 5. The vector of natural demands \((x_i(p))_{i \in I}\) is a Nash equilibrium under the NDC, i.e. \(\tilde{x}_i(p) = x_i(p)\), for all prices \(p \geq p^{**}\).

Proof. We want to prove that at prices above the second best \(p^{**}\) individual demands under the NDC coincide with natural demands without profit sharing, that is \(\tilde{x}_i(p) = x_i(p)\). Recall that by definition \(p^{**}\) is such that \(p^{**} = AC(x(p^{**}))\) \((x(p) \equiv \sum_i x_i(p))\). Since utility functions under the NDC are affected by the slope properties of \(AC(\cdot)\), their general formal definition is rather complicated. To avoid an unnecessarily heavy notation, in this and the following proof we restrict ourselves to the case of global diseconomies of scale (everything, however, smoothly carries over to the case of economies of scale, after suitably adjusting the definitions).

Given \(p \geq p^{**}\) and \(x_{-i}(p)\), the individual utility under the NDC is defined as follows:

\[
V(x_i, x_{-i}(p), \theta_i) = \begin{cases} u(x_i, \theta_i) - px_i, & 0 \leq x_i \leq AC^{-1}(p) - x_{-i}(p), \\ u(x_i, \theta_i) - px_i + \frac{p(x_i + x_{-i}(p))}{n}, & AC^{-1}(p) - x_{-i}(p) < x_i \leq AC^{-1}(p) - x_{-i}(p), \\ C(x_i + x_{-i}(p)), & x_i > AC^{-1}(p) - x_{-i}(p). \end{cases}
\]

Utility is increasing on \([0, x_i(p)]\), since \(x_i(p) < AC^{-1}(p)\) and therefore coincides with the natural utility (see the upper line of the previous equation). Instead, for \(x_i > x_i(p)\) we have \(V(x_i, x_{-i}(p), \theta_i) \leq V(x_i(p), x_{-i}(p), \theta_i)\) (note that beyond \(AC^{-1}(p)\) a negative profit share is associated to a lower consumer surplus). Then \(\arg\max_{x_i} V(x_i, x_{-i}(p), \theta_i) = x_i(p)\) and \((x_i(p))_{i \in I}\) is a Nash equilibrium at \(p\). \(\Box\)

Lemma 6. At prices \(p \in (\hat{p}^{**}, p^{**})\) there exist Nash demand vectors under the NDC, \((\tilde{x}_i(p))_{i=1}^n\), all of which are characterized by the property \(p = AC(\tilde{x}(p))\), \(x(p) \equiv \sum_i \tilde{x}_i(p)\).

Proof. Consider first the case of diseconomies of scale. Without loss of generality we assume that there exists a unique Nash demand function under profit-sharing, \((\hat{x}_1(p), \hat{x}_2(p), \ldots, \hat{x}_n(p))\).\(^{25}\) We adopt the following notational conventions: for any vector \(Z = (z_1, z_2, \ldots, z_n)\) we define \(z \equiv \sum_i z_i\), \(z_{-i} \equiv \sum_{h \neq i} z_h\). For given

\(^{25}\)Allowing for multiple Nash demand vectors, \((\hat{x}_i^a(p))_{i=1}^n, (\tilde{x}_i^a(p))_{i=1}^n\), would add nothing substantial, since it is enough to set \(\hat{x}_i(p) \equiv \max \{ \hat{x}_i^a(p), \tilde{x}_i^a(p), \ldots \}\).
\( p \) and \( X_{-i} \equiv (x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n) \) individual \( i \)'s objective function (3) is specified under the NDC as follows

\[
V(x_i, X_{-i}, \theta_i) = \begin{cases} 
\begin{align*}
 u(x_i, \theta_i) &= px_i, \\
 u(x_i, \theta_i) &= px_i + \frac{p(x_i + x_{-i})}{n} - \frac{C(x_i + x_{-i})}{n},
\end{align*}
\end{cases}
\]

\( x_{-i} = \sum_{h \neq i} x_h \). We ask if at any price \( \bar{p}, \hat{p} < \bar{p} < p^{**} \), there exists a Nash demand vector \( (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n) \), \( \hat{x}(\bar{p}) < \bar{x} < x(\bar{p}) \), \( \bar{x} = \sum_i \bar{x}_i \) (outside interval \([\hat{x}(\bar{p}), x(\bar{p})]\) there is obviously no equilibrium). Note that the total quantity that solves equation \( \bar{p} = AC'(x) \) belongs to \([\hat{x}(\bar{p}), x(\bar{p})]\) by the continuity of \( AC \). There are three cases to be examined: A) \( \bar{x} < AC^{-1}(\bar{p}) \), B) \( \bar{x} > AC^{-1}(\bar{p}) \), C) \( \bar{x} = AC^{-1}(\bar{p}) \) \((AC(\cdot)\) is invertible by the monotonicity property).

A) \( \bar{x} < AC^{-1}(\bar{p}) \). Profits are positive but do not accrue to members, owing to the NDC. Then, since \( \hat{x}_i(\bar{p}) < \bar{x}_i < x_i(p) \) for at least one \( i \), there exists a demand \( x'_i \), \( \bar{x}_i < x'_i < AC^{-1}(\bar{p}) - \bar{x}_{-i} \), such that

\[
\begin{align*}
 u(x'_i, \theta_i) - \bar{p}x'_i &> u(\bar{x}_i, \theta_i) - \bar{p}\bar{x}_i \\
 x_i > AC^{-1}(p) - x_{-i},
\end{align*}
\]

and therefore \( \bar{x}_i \) is not an equilibrium demand.

B) \( \bar{x} > AC^{-1}(\bar{p}) \). In this case at \((\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)\) there are realized and then distributed losses, since loss distribution is not prevented by the NDC. In order for \((\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)\) to be a Nash equilibrium at \( \hat{p} \) with loss-sharing, there must hold

\[
\begin{align*}
 u_1(\bar{x}_i, \theta_i) - \bar{p} + \frac{\bar{p}}{n} - \frac{C'(\bar{x}_i + \bar{x}_{-i})}{n} &= 0.
\end{align*}
\]

for at least one \( i \). Then, since \( \hat{x}(\bar{p}) < \bar{x} \), by the convexity of \( C(\cdot) \) there also holds

\[
\begin{align*}
 u_1(\bar{x}_i, \theta_i) - \bar{p} + \frac{\bar{p}}{n} - \frac{C'(\bar{x}_i + \hat{x}_{-i}(\bar{p}))}{n} &> 0. \quad (12)
\end{align*}
\]

Recalling that \( \hat{x}_i(p) \) is an unconstrained optimum with profit-sharing, we can also write

\[
\begin{align*}
 u_1(\bar{x}_i(\bar{p}), \theta_i) - \bar{p} + \frac{\bar{p}}{n} - \frac{C'(\bar{x}_i(\bar{p}) + \hat{x}_{-i}(\bar{p}))}{n} &= 0. \quad (13)
\end{align*}
\]

By the concavity of the objective function \( V(\cdot, \theta_i) \) from (13) there follows

\[
\begin{align*}
 u_1(\bar{x}_i, \theta_i) - \bar{p} + \frac{\bar{p}}{n} - \frac{C'(\bar{x}_i + \bar{x}_{-i})}{n} &< 0.
\end{align*}
\]
which contradicts (12) and hence \((\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)\) is not a Nash demand vector.

C) \(\bar{x} = AC^{-1}(\bar{p})\). Consider a \((\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)\) such that \(\hat{x}_i(\bar{p}) < \bar{x}_i < x_i(\bar{p})\).

Since objective functions are not differentiable at any \((\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)\) such that \(\bar{x} = AC(\bar{p})\), we need to specify left-hand and right-hand optimum conditions. Given \(\bar{x}_-\), on \([0, \bar{x}_i]\) there are realized profits which are retained by the firm to comply with the NDC. Therefore, the left-hand first order condition for member \(i\) at \(\bar{x}_i\), \(u_1(\bar{x}_i, \theta_i) > \bar{p}\), is satisfied, since \(u(\cdot, \theta_i)\) is concave and, by the definition of \(\hat{x}_i(\bar{p})\), there also holds \(u_1(x_i(\bar{p}), \theta_i) = \bar{p}\). (Thanks to the maximand’s concavity the FOC is sufficient too and hence \(u(\bar{x}_i, \theta_i) - \bar{p}\bar{x}_i > u(x_i, \theta_i) - \bar{p}x_i, x_i < \hat{x}_i\).

Instead, given \(\bar{x}_-\), there arise losses on the right-hand interval \([\bar{x}_i, x_i(\bar{p})]\), which are distributed among members and the utility function is thus defined as

\[
V(x_i, X_{-i}, \theta_i) \equiv u(x_i, \theta_i) - \bar{p}x_i + \frac{p(x_i + \bar{x}_{-i}) - C(x_i + \bar{x}_{-i})}{n}
\]

whose right-hand first order condition

\[
u_1(\bar{x}_i, \theta_i) - \bar{p} + \frac{p}{n} - \frac{C'(\bar{x}_i + \bar{x}_{-i})}{n} < 0.
\]

is satisfied by the concavity of \(V(\cdot, X_{-i}, \theta_i)\). Then, any \((\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n)\) such that \(\hat{x}_i(\bar{p}) < \bar{x}_i < x_i(\bar{p})\), and \(\bar{x} = AC^{-1}(\bar{p})\), \(\bar{x} = \sum_i x_i\), is a Nash equilibrium at \(\bar{p}\).

An analogous conclusion can be obtained along the same lines for the case of economies of scale (omitted). \(\square\)

**Lemma 7.** Inequality \(p_i^* \geq \langle < \rangle p^*\) holds if and only if

\[
\frac{\hat{x}_i(p_i^*)}{\bar{x}(p_i^*)} \leq (>) \frac{1}{n}.
\]

**Proof.** Let us rewrite condition (7) for \(p = p_i^*\),

\[
\left[ \frac{\hat{x}(p_i^*)}{n} - \hat{x}_i(p_i^*) \right] + \frac{\hat{x}(p_i^*)}{n} \left[ p_i^* - C'(\hat{x}(p_i^*)) \right] = 0.
\]

Since \(\hat{x}'(p) < 0, \forall p\), the sign of the first difference in brackets is equal to that of the second one, i.e.

\[
p_i^* \geq \langle < \rangle C'(\hat{x}(p_i^*)) \Leftrightarrow \frac{1}{n} \geq \langle < \rangle \frac{\hat{x}_i(p_i^*)}{\bar{x}(p_i^*)}.
\]

We know that at first-best prices the sharing effect is null, as \(p^* = C'(x(p^*)) = C'(\hat{x}(p^*))\), and the curves \(x(p)\) and \(\hat{x}(p)\) cross at it, \(x(p^*) = \hat{x}(p^*)\) (Lemma 2).
By the strict concavity of the social surplus there holds $p(x) > (\leq,\geq) C'(x)$, for $x < (>) x^* (p(x) \equiv x^{-1}(x))$ and the first-best price $p^*$ is unique. Note that the effective demand-price function $\hat{p}(x)$ is such that $\hat{p}(x) \geq (\leq) p(x) \geq (\leq) C'(x)$ for $x \leq (>) x^* (\hat{p}(x) \equiv \hat{x}^{-1}(x))$. Therefore, when there occurs profit-sharing, there holds

$$p \geq (\leq) C'(\hat{x}(p_i^*)) \iff p \geq (\leq) p_i^*.$$  

for any $p$. We are then allowed to replace $p_i^* \geq (\leq) C'(\hat{x}(p_i^*))$ with $p_i^* \geq (\leq) p_i^*$ in (14) and this completes the proof. \hfill \Box

**Lemma 8.** When the median member $m$ is such that $m < (>) a$, switching from the first-best price to $m$’s own preferred one $p_m^*$ causes the utility of all $i \leq (\geq) m$ to increase and that of all $i > (>) m$ to decrease.

**Proof.** We first prove that $p_i^*$ is monotonic decreasing in $i$. By the strict concavity in $p$, functions $U(p, \theta_i)$ have a single maximizer $p_i^* \equiv \arg\max_p U(p, \theta_i)$ for each $\theta_i$, such that

$$-\hat{x}(p_i^*) + \frac{1}{n} [p_i^* \hat{x}'(p_i^*) + \hat{x}(p_i^*) - C'(\hat{x}(p_i^*))\hat{x}'(p_i^*)] = 0$$

($\hat{x}_i, \hat{x}$ are defined at Lemma 2). Take an $\hat{x}_j(p_i^*) > \hat{x}_i(p_i^*)$. By Lemma 3 and the strict monotonicity of $\theta_j$ in $i$, there holds $j > i$ and by replacing $\hat{x}_i(p_i^*)$ with $\hat{x}_j(p_i^*)$ in the previous equation we get

$$\left[\frac{\hat{x}(p_j^*)}{n} - \hat{x}_j(p_i^*)\right] + \frac{\hat{x}'(p_i^*)}{n} \left[p_i^* - C'(\hat{x}(p_i^*))\right] < 0, j > i,$$

whence in turn there follows $p_j^* < p_i^*$ by the strict concavity of $U(p, \theta_i)$.

Now let us focus on the case $m < a (p_m^* > p^*)$. Take any $i < m$. Since $p_i^* > p_m^* > p^*$, $U(p, \theta_i)$ is increasing at both $p^*$ and $p_m^*$, as represented in figure 5. Then, $U(p_m^*, \theta_i) > U(p^*, \theta_i), \forall i < m$. Conversely, for $i > m$ utility $U(p, \theta_i)$ is decreasing at both $p^*$ and $p_m^*$ and hence $U(p_m^*, \theta_i) < U(p^*, \theta_i), \forall i > m$. The proof for the case $m > a (p_m^* < p^*)$ follows the same lines and is omitted. \hfill \Box
Figure 5 – Optimal prices for $i, m, i < m < a$

References


